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Competitive Swim Trunks or Competitive Prices?

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Abstract—*Speedo has released many series of competitive racing trunks, which supposedly provide improved racing performance and timings in the pool. The material used in the competitive swim trunks is different to the material used in regular swim trunks.*

The research question that this investigation aims to answer is:
Do Speedo's LZRs provide a definite improvement in competitive swimming in terms of lower drag force acting on the wearer?

In order to conduct this test, material from both regular swim trunks and LZR racing swim trunks was obtained. A formula was derived from Raleigh's Drag Law, and balls wrapped with these materials were dropped in water over time. Using tracking software, this motion was tracked, and based on the data obtained, the drag constant acting on each material was determined.

It can be concluded from the experiment that the material of Speedo's LZR racing trunks does in fact have less drag acting on it in water than the training swim trunks do.

1. INTRODUCTION

A distinction can be made between the two different types of swimming trunks. There are trunks for racing (competitive swim trunks) and trunks for training.

Speedo's first series of the "LZR racing trunk" created a buzz before and during the 2008 Beijing Olympic Games, where an astonishing 62 world records[1]were broken in a 4 month period by swimmers wearing this new racing trunk. Eventually, after rival manufacturers such as Arena and TYR filed a number of cases against the trunks, these trunks were banned by FINA (the international body that governs competitive swimming), as they were considered to give too unfair an advantage to the swimmer. The trunkswere said to have an extremely lightweight design and were made of optimal fabric quality[2]that vastly increased hydrodynamics and reduced drag in the water. This drag reduction was also assisted by the water repellent nature of the trunks. At that time, the LZRswere priced at more than $400.

Speedo developed new swimming trunks, following FINA's new guidelines, and released 2 new racing series. These are the "LZR X" racing series and the Speedo "Fastskin LZR" series, both of which gained FINA approval. These trunks both also offer great hydrodynamics and are said to be water repellent. The hydrophobic (water repelling) material assists in reducing the drag on the swimmer. These suits are now priced at 260 euros and 185 euros[4], respectively. In contrast a typical pair of training trunks only costs around 42 euros.

While with my school swim team, I noticed that this trend was not lost here either. Swimmers donning the Speedo LZRs tended to have slight advantage when it came to competitive races, even if they were on par with their teammates during training. Based on this, I decided to conduct the following investigation.

2. AIM OF THE INVESTIGATION:

The aim of this experiment was to **find out whether Fastskin LZR racing trunks offered a significant advantage over the training trunks and hence justified their price tag.**

To carry out this investigation, a derivation of a velocity function from Raleigh's drag law was used. This helped to compare the drag coefficient between the two pairs of trunks to determine which was more efficient in the water and whether the racing trunks offered a significant difference over the non-racing trunks.

Firstly, the velocity function was derived by the use of differential equations. Then, I cut out a part of each pair of swimming trunks and sewed it around a spherical ball. The use of actual material from an old pair of Fastskin LZR racing trunks and an old pair of training trunks made this experiment more realistic. The use of a spherical ball was necessary. This ensured that the law of drag remained as it should. This law only applies when objects of spherical shape flow in water.

Each ball was made of a dense material and had a mass of 245g. Each ball (one wrapped in training trunk material and the other wrapped in Fastskin LZR racing trunk material) was dropped in water.

A video was taken of the balls falling in water, making use of an iPhone camera (in a waterproof case) underwater. All videoing took place at 120fps (frames per second). The recording was carried out in a pool. Firstly, this reduced the effect of walls on the force of drag. This effect becomes too large in a narrow container. Secondly, the pool, being about 1.5m deep was of sufficient depth to ensure that the balls reached terminal velocity.

The videos were inserted into a video tracking software called "Tracker", which produced data about the velocity of the ball with the passage of time. This data produced a graph that was inverted exponential in shape and the graph was fit to the velocity equation derivation from Raleigh's drag law using the graphing software "Logger Pro".

Finally, this allowed the drag constant to be determined according to the constants of the graph curve fit, and a comparison between the drag force of the two different types of trunks could be made.

3. RALEIGH'S DRAG LAW

Raleigh's law of drag is as follows:

$$F_d = \frac{1}{2} C_d p v^2 A$$

F_d refers to drag force,

C_d refers to a dimensionless drag coefficient,

p refers to density of the medium,

v refers to flow velocity and

A refers to cross sectional area.

3.1 Derivation of a formula for velocity:

To simplify the equation, the constants were grouped together. It was possible to do this as the constants were the same for both swimsuits and trials.

Therefore, since $F_d \propto -v^2$, we can say that $F_d = -Cv^2$, where *C* represents the grouped constants. A change in the variable *C* would hence affect the drag force and this was treated as a new drag constant.

The drag force acting on the ball was now clarified in the context of the investigation.

The diagram shows the forces acting on the ball inside a swimming pool.

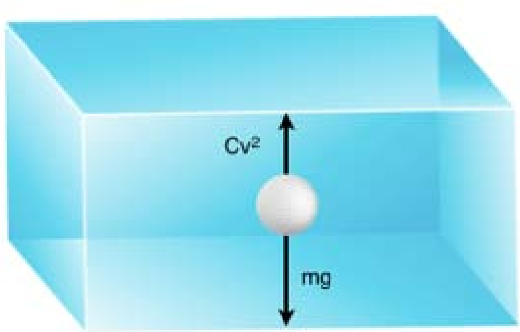


Fig. 1: Ball failing in water

It is accepted that $F = ma$, where F is the force on an object, m is the mass of an object and a is the acceleration of the object.

It is also accepted that the downward force on the ball is that of its weight, $W=mg$, where W is theweight of the ball, m is themass of the ball and g is theacceleration of gravity (9.8m/s^2)

Therefore, the resultant force on the ball = $mg - Cv^2 = ma$

$a$was treated as the differential of velocity, hence forming the equation $mg - Cv^2 = m\frac{dv}{dt}$

Rearranging the equation:

$$\frac{mg}{C} - v^2 = \frac{m}{C}\frac{dv}{dt}$$

$$\Rightarrow \frac{C}{m}dt = \frac{1}{\frac{mg}{C} - v^2}dv$$

$$\Rightarrow \frac{C}{m}\int_0^t dt = \int_{v_0}^{v_f} \frac{1}{\frac{mg}{C} - v^2}dv$$

, where$t$ refers to time, $v_f$refers to final velocity at any time t and v_0 refers to initial velocity.

A differential equation was now in place, which allowed a derivation of the velocity function.

To solve this equation, **the integral on the right hand side** must first be dealt with. This takes the form of $\int \frac{dy}{u^2 - y^2}$, where $u = \sqrt{\frac{mg}{C}}$ and $y = v$

This integral can be solved as follows:

$\frac{1}{u^2 - y^2}$ can be split into two fractions where $\frac{1}{u^2 - y^2} = \frac{A}{u+y} + \frac{B}{u-y}$, so that integration by parts can be carried out.

$$\Rightarrow A(u-y) + B(u+y) = 1$$

$$\Rightarrow Au - Ay + Bu + By = 1$$

$$\Rightarrow -A + B = 0$$

(Since no value of u makes an appearance)

$$and\, A + B = \frac{1}{u}$$

$$\Rightarrow A\, and\, B\, both = \frac{1}{2u}$$

This can also be seen by an identity of substituting in any value of u.

The integral can now be more easily solved, while still keeping it in the form $\frac{du}{u^2 - y^2}$

$$\int \frac{dy}{u^2 - y^2} = \int \frac{dy}{2u(u+y)} + \int \frac{dy}{2u(u-y)}$$

$$= \frac{1}{2u}\left(\int \frac{dy}{u+y} + \int \frac{dy}{u-y} \right)$$ (the common factor is brought out)

$$= \frac{1}{2u}\left[\ln|u+y| + (-\ln|u-y|) \right]$$

$$= \frac{1}{2u}\left(\ln\left|\frac{u+y}{u-y}\right| \right)$$

Values from the original integral can now be substituted back in. These are $u = \sqrt{\frac{mg}{C}}$ and $y = v$.

$$\Rightarrow \int_{v_0}^{v_f} \frac{1}{\frac{mg}{C} - v^2} dv = \frac{1}{2\sqrt{\frac{mg}{C}}}\left[\ln\left| \frac{\sqrt{\frac{mg}{C}} + v}{\sqrt{\frac{mg}{C}} - v} \right| \right]_{v_0}^{v_f} = \frac{1}{2\sqrt{\frac{mg}{C}}}\left[\ln\left| \sqrt{\frac{mg}{c}} + v \right| - \ln\left[\sqrt{\frac{mg}{c}} - v \right] \right]$$

$$= \frac{1}{2\sqrt{\frac{mg}{C}}}\left[\ln\left| \frac{\left(\sqrt{\frac{mg}{C}} + v_f\right)\left(\sqrt{\frac{mg}{C}} - v_0\right)}{\left(\sqrt{\frac{mg}{C}} - v_f\right)\left(\sqrt{\frac{mg}{C}} + v_0\right)} \right| \right]$$

To simplify this, v_t(terminal velocity) can be equated to $\sqrt{\frac{mg}{C}}$, as the terminal velocity is essentially a function of the mass multiplied by the gravitational force, divided by the constants.

Now, we can reconsider the original differential equation:

$$\frac{C}{m}\int_{v_0}^{t} dt = \int_{v_0}^{v_f} \frac{1}{\frac{mg}{C} - v^2} dv$$

$$\frac{Ct}{m} = \frac{1}{2v_t} \ln\left| \frac{(v_t + v_f)(v_t - v_0)}{(v_t - v_f)(v_t + v_0)} \right|$$

$$\frac{Ct}{m} \times 2v_t = \ln\left| \frac{(v_t + v_f)(v_t - v_0)}{(v_t - v_f)(v_t + v_0)} \right|$$

$$\Rightarrow \frac{(v_t + v_f)(v_t - v_0)}{(v_t - v_f)(v_t + v_0)} = e^{\frac{2Ctv_t}{m}}$$

This equation now has to be rearranged in terms of v_f

$$\frac{(v_t + v_f)}{(v_t - v_f)} = \frac{(v_t + v_0)}{(v_t - v_0)} e^{-\frac{2Ctv_t}{m}}$$

We can let $\frac{(v_t + v_0)}{(v_t - v_0)}$ = n

The fraction $\frac{(v_t + v_f)}{(v_t - v_f)}$ can be split into $-1 + \frac{2v_t}{(v_t - v_f)}$

$$\Rightarrow \frac{2v_t}{(v_t - v_f)} = 1 + ne^{\frac{2Ctv_t}{m}}$$

$$v_t - v_f = \frac{2v_t}{1 + ne^{\frac{2C_tv_t}{m}}}$$

$$\Rightarrow v_f = v_t - \frac{2v_t}{1 + ne^{\frac{2Ctv_t}{m}}}$$

The velocity function was thus obtained from Rayleigh's drag law.

4. EXPERIMENTATION:

The experimentation aspect of this investigation was then carried out.

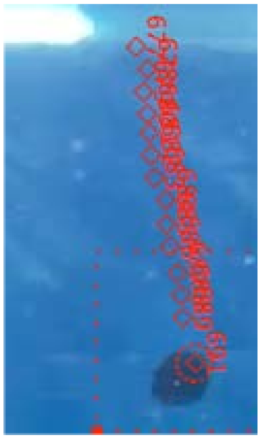
2 dense balls weighing 245g were each covered with material from each type of swimsuit. The material was stretched and sewn onto the ball to ensure tightness when being wrapped around the ball, to simulate the tightness of swimming trunks on a swimmer's body.

The two balls were then dropped into water, one at a time. This was videoed underwater at 120 frames per second for each ball.

The "Tracker" software gave frame by frame data for the movement of the balls.

Below are screenshots of the images from tracker software:

Training Trunks



Racing Trunks

The movement of the ball was noted by the “Tracker” software, using an auto-tracking device, within the software itself. For both balls, the “Tracker” software used the data plots to obtain a data set for the velocity of the balls with the passage of time, using arbitrary units for velocity.

The data points were all recorded and inserted into the “Logger Pro” software,which created a scatter plot.

The graphs produced allowed for better analysis as they were produced in the form of the velocity equation hence allowing us to find the value of the constant C and gain a better understanding of the drag force on each ball.

It is important to note that the balls drifted slightly to one side. Although they both drifted towards the same side, it is possible that this could have had some effect on the graphs obtained. However, this was difficult to control as it was likely to do with currents in the water or the shape of the material around the ball. Attempts were also made to control the height from which the ball was released in the water by releasing the ball from the surface each time, though this was not as significant since the water was deep enough to ensure that the ball would reach terminal velocity.

4.1 Data:

Time (s)	Velocity (AR)
0	0
0.03334	128.1318
0.066673	242.2735
0.100006	365.7484
0.13334	269.3124
0.166673	245.8950
0.200006	326.2032
0.23334	265.6145
0.266673	293.6519
0.300006	305.9970
0.33334	313.8041
0.366673	302.9570
0.400006	319.7042
0.43334	330.5796
0.466673	300.7011
0.500006	297.4030
0.53334	296.4089
0.566673	304.9666
0.600006	326.5212
0.63334	323.3265
0.666673	307.4118
0.700006	324.0053
0.73334	320.8763
0.766673	301.1060
0.800006	307.3551
0.83334	311.8114
0.866673	298.2825
0.900006	290.1289
0.93334	301.4901
0.966673	307.9707

Fig. 2. Screenshot of Velocity-time data for ball with training trunkswrapped around it.

Time (s)	Velocity (Arbitary)
0	0
0.033333	126.1252
0.066666	240.9070
0.1	280.1030
0.133333	285.2985
0.166666	260.8418
0.2	330.0693
0.233333	285.6476
0.266666	305.7196
0.3	300.4438
0.333333	358.4297
0.366666	340.8714
0.4	309.2145
0.433333	308.7944
0.466666	320.0662
0.5	298.5152
0.533333	303.6805
0.566666	311.5253
0.6	302.3242
0.633333	313.2827
0.666666	308.8750
0.7	286.3156
0.733333	289.0385
0.766666	308.2877
0.8	314.0120
0.833333	308.5798
0.866666	307.3661
0.9	304.6323
0.933333	306.2509
0.966666	299.9371
1	288.3017

Fig. 3. Screenshot of Velocity-time data for ball with racing trunkswrapped around it.

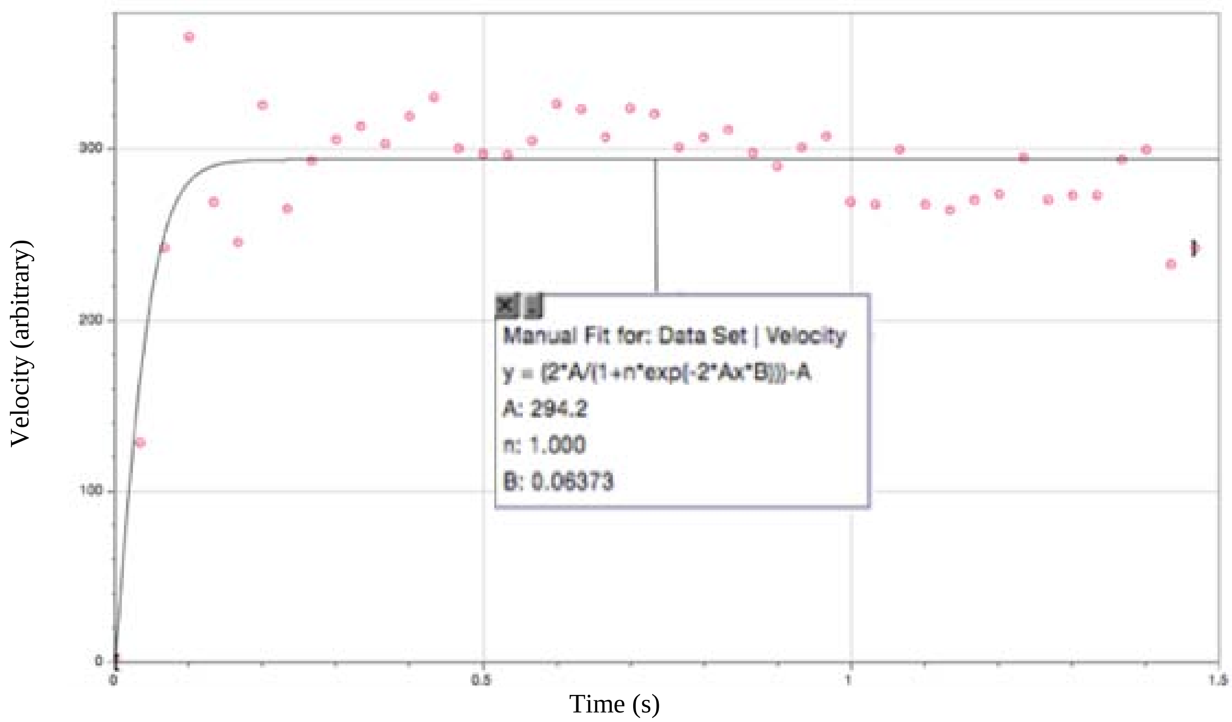
Using the above data for each ball, a graph of the results was plotted. The inverted exponential curve was fitted according to the equation $f(t) = \frac{2A}{1+ne^{-2ABt}} - A$ in order to represent the form of the velocity function derived previously.

A represents v_t, *n* represents the constant *n* as in the previous equation and *B* represents the constant value of $\frac{C}{m}$ for each graph.

By fitting a curve of the data to the above generalized equation in a software known as "Logger Pro", the aim was to obtain the value of B for each curve fit. Obtaining this value would allow calculation of the value of C since the value of m (mass of 245g) was already known.

Comparing the x values allowed determination of which swimsuithad a greater drag force, as the trunks with the greater C value would have a greater drag force, due to $F_d \propto Cv^2$

4.2 Graphs:



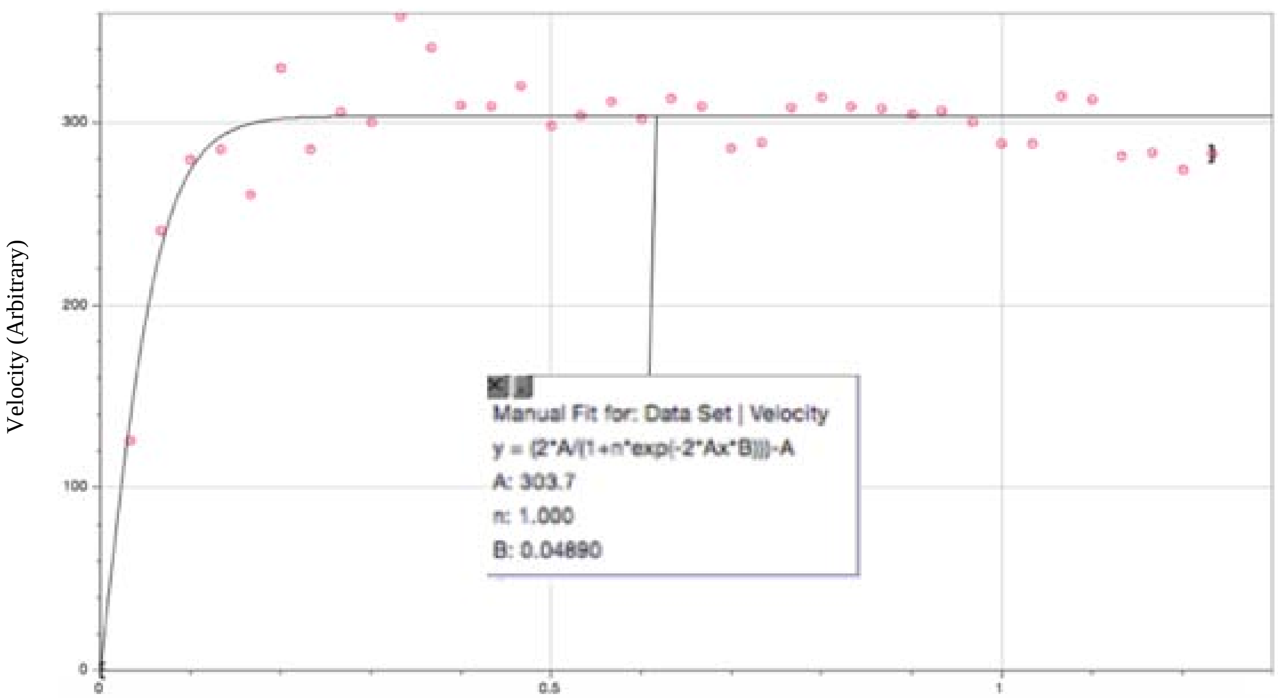
Graph 1: velocity with passage of time for ball wrapped with training trunks

The above shows the Logger Pro curve fit for the ball wrapped with training trunks. As can be seen from the curve fit, the terminal velocity value is 294.2 arbitrary units and the obtained value of *B* from the fit is 0.06374. This means that the value of drag constant C obtained from this pair of swimming trunks is $0.06374 \times m$, where m is 245g (the mass of the ball).

Therefore, for the training trunks, drag constant $C = 15.6(3s.f)$

Arbitrary units are used in both graphs 1 and 2 because they function simply as a comparison between the two.

Graph 2: velocity with passage of time for ball wrapped with Fastskin LZR racing trunks



This graph shows the Logger Pro curve fit for the ball wrapped in Fastskin LZR racing trunks. It can be seen that according to the curve fit, the Fastskin LZR racing trunks have a terminal velocity of 303.7 arbitrary units. The graph fit also gives the *B* value to be 0.04890. Once again the value of the drag constant C for these swim trunks is

$= 0.04890 \times m$ (where*m* = 245g, which was the mass of the ball used).

Therefore, for the racing trunks, drag constant $C = 12.0\,(3SF)$

5. CONCLUSION

It can be seen from the graphs obtained and the values of the constants obtained from the curve fits that the racing trunks do offer a significant advantage in the water. Firstly, the drag constant is significantly reduced at 12.0 when the Fastskin LZR racing trunks are used compared to 15.6 when training trunks are used. This in turn would mean that the drag force that acts upon the Fastskin LZR racing trunks is significantly lower than the drag acting upon the training trunks, likely due to the material and water repellant properties of the racing trunks. This would help increase the speed of a swimmer in water.

Apart from this however, the racing trunks also allow a higher terminal velocity to be reached (303.7 as opposed to the 294.2 arbitrary units reached by the training trunks). This means that the swimmer is also able to accelerate more in water and this can make a difference in this sport, where every millisecond counts, thereby assisting swimmers to have an edge over their competitors in sprints and perhaps offering more significant advantages in a longer race format.

This investigation would not be complete without highlighting some limitations. A spherical ball had to be used in order to ensure the laminar flow that the law of drag is limited to. The actual body of a swimmer is shaped quite differently from a ball. The experiment was also conducted in a pool rather than a clear large cylindrical container as a certain distance from the wall was necessary for the laws to remain as they are. The effect of the wall on the movement of the ball slightly skews Rayleigh's drag law. However, for a repeat of this experiment, it would be more ideal to use a wide clear cylindrical container, with a diameter of around 30cm. This would prevent any other interference but would also ensure that the walls do not interfere with the law. It is also important to note that old samples of both kinds of trunks were used for the experiment to save on high cost. This may have had some impact since some of the properties of both trunks may have been lost with usage over time. Finally it is important to note that the data points obtained are uneven and shaky. This is likely due to the quality of the underwater video as well as some shaking of the hand and these uneven points are to be expected. It would have been ideal to mount the camera on a tripod underwater to create stability and prevent any movement. This would be done in any future experimentation. However, despite the fluctuation of the points, the general trend obtained is more important.

Overall, I felt that the method used to generate a velocity-time graph of the fall of both balls was quite reasonable and gave an acceptable comparison of the drag force of each type of swim trunk.

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